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The (BFKL) Pomeron-γ^*-γ vertex for any conformal spin

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Abstract. To study diffractive photon production at HERA, we compute the projection of the $\gamma^*\gamma$ impact-factor on the BFKL leading-order eigenfunctions $E^{n,\nu}$ for non-zero transfer. This calculation supplements former ones performed for $n = 0$. We provide an expression for $n = \pm 2$ and check that all the other components are zero.

1 Introduction

The BFKL equation has been widely used to study the inclusive or semi-inclusive observables measured at HERA (structure functions, diffractive structure functions) in the small-x (large s) kinematical region [1]. It is an evolution equation for the gluon density and was written and solved at leading-log$(1/x)$ level (LLx) [2–4] and at next-to-leading accuracy (NLLx) [5,6].

In the following, we will be concerned with the LLx kernel, which exhibits an interesting property: in the space of the transverse positions ρ_1 and ρ_2 of the evolved gluons, it was shown to be invariant under the global conformal transformations [7]. Hence the general solution of the BFKL equation can be written as a sum over the kernel eigenfunctions corresponding to the irreducible representations of the symmetry group $SL(2,\mathbb{C})$. The latter are indexed by two indices, one of them being the (discrete) conformal spin n, the other one the (continuous) real parameter ν. The dominant energy behaviour is a power-like rise of the amplitude $s^{\Delta_{\mathbb{P}}}$, where the intercept value $\Delta_{\mathbb{P}} \approx 0.3 - 0.5$ usually quoted is given by the $n = 0$ component.

However, the phenomenological relevance of the higher-spin components ($n \neq 0$) was suggested in [8]. In that paper, it is shown that the $n = \pm 2$ components appearing through the BFKL resummation can mimic the *soft pomeron*. Indeed, this component exhibits effectively the right energy dependence of the soft pomeron and reproduces well its "higher-twist" behaviour at moderate and large Q^2 pointed out by Donnachie and Landshoff [9]. Nevertheless, a number of points of the conjecture were left untested. In particular, only a phenomenological expression for the coupling of the photon impact-factor to the higher-spin components was used. Moreover, the non-forward behaviour was not considered for lack of a precise knowledge of the relevant coupling at the photon vertex.

Recently, much efforts have been devoted to the study of the photon impact-factor in the transition $\gamma^* \to \gamma$ [10, 11]. Its coupling to the LLx BFKL pomeron was computed but only for $n = 0$. We propose in this paper to extend these calculations to $n = \pm 2$. The odd components are automatically zero by symmetry. Furthermore, we have checked that the $|n| > 2$ ones are zero. As we will see later, this is due to the fact that the coupling of two spin-1 photons selects conformal spins smaller or equal to 2. We provide the explicit expressions for $n = 0$ and $n = \pm 2$ (see (3.28) and subsequent equations).

The general framework of our calculation is presented in Sect. 2. The calculation appears in some details in Sect. 3. To keep it readable, the most technical point are outlined in the five appendices **A-E** where we show in particular the evaluation of a generalized hypergeometric ${}_3F_2$ function. Finally, we draw our conclusions and suggest outlooks.

2 Definition of the scattering process

We consider the scattering of the two objects 1 and $1'$ into 2 and $2'$ (see Fig. 1). The functions $\phi(\rho_1, \rho_2)$ (resp. ϕ') are their impact factors and depend on the bidimensional variables ρ_1, ρ_2 which are conjugate to the transverse momenta $\mathbf{k}$, $\mathbf{k} + \mathbf{q}$ of the exchanged gluons. We deal with an impact factor involving a virtual photon in the initial state and a real one in the final state. If furthermore the object $1'$ is a proton, this process models diffractive photon production at HERA with a large rapidity gap generated by the BFKL resummation. If $1'$ is also a photon, we have access to $\gamma^*\gamma^*$ physics. We should not enter into these details in this paper, and we will only stick to the $\gamma^* \to \gamma$ impact-factor.

In the BFKL framework, the most general expression for the scattering amplitude of two objects 1 and $1'$ reads [7]:

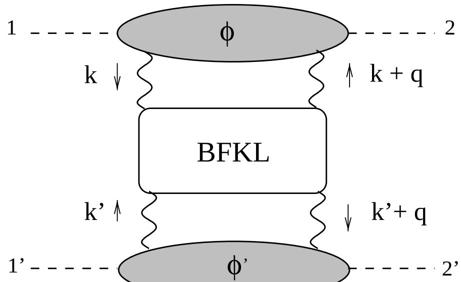

Fig. 1. Scattering process. 1 and 2 label the colliding objects. The wavy lines represent exchanged reggeized gluons which interact through the BFKL kernel. The arrows give the momentum flux

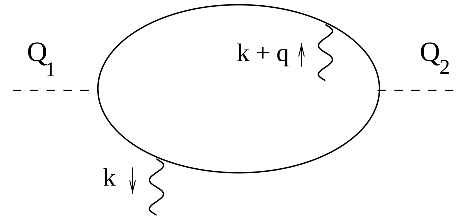

Fig. 2. Diagram contributing to the impact factor ϕ_1. The dashed lines represent (virtual or real) photons which resolve into a quark-antiquark pair. The wavy lines stand for off-shell gluons. Four such diagrams (corresponding to each possible insertion of the two exchanged gluons) have to be taken into account although (see text) only two of them effectively contribute to the amplitude

$$A(s,t) = \frac{i\,s\,G}{(2\pi)^{10}} \sum_{n=-\infty}^{+\infty} \int d\nu \frac{\nu^2 + \frac{n^2}{4}}{\left(\nu^2 + \left(\frac{n-1}{2}\right)^2\right)\left(\nu^2 + \left(\frac{n+1}{2}\right)^2\right)} \times e^{\bar{\alpha}\log(s/s_0)\chi_n(\nu)}\; I^{n,\nu}\; \bar{I}'^{n,\nu}\ , \tag{2.1}$$

where G is the appropriate colour factor corresponding to the process under consideration, $\bar{\alpha} \equiv \alpha_s N_c/\pi$ and $\chi_n(\nu)$ is the well-known eigenvalue of the leading-order BFKL kernel:

$$\chi_n(\nu) = 2\Psi(1) - 2\,\mathcal{R}\mathrm{e}\Psi\left(\frac{1+|n|}{2} + i\nu\right)\ . \tag{2.2}$$

The scale s_0 is undetermined at LLx. The functions $I^{n,\nu}$ and $\bar{I}'^{n,\nu}$ are the "vertex functions", i.e. the impact factors ϕ projected on the corresponding eigenfunctions $E^{n,\nu}$ of the BFKL kernel:

$$I^{n,\nu} = -\frac{1}{4}\int d\rho_1 d\bar{\rho}_1 \int d\rho_2 d\bar{\rho}_2 \phi(\rho_1,\rho_2) E^{n,\nu}(\rho_1,\rho_2)\ , \tag{2.3}$$

where $\phi(\rho_1,\rho_2)$ is the impact-factor, and

$$E^{n,\nu}(\rho_1,\rho_2) = (-1)^n \left(\frac{\rho_1-\rho_2}{\rho_1\rho_2}\right)^a \left(\frac{\bar{\rho}_1-\bar{\rho}_2}{\bar{\rho}_1\bar{\rho}_2}\right)^{\tilde{a}}\ . \tag{2.4}$$

The convenient notations $a = \frac{1-n}{2} + i\nu$ and $\tilde{a} = \frac{1+n}{2} + i\nu$ have been introduced in the previous equation. We work with complexified transverse vectors $\rho = \rho_x + i\rho_y$ and $\bar{\rho} = \rho_x - i\rho_y$.

The amplitude A is invariant by rotation. The BFKL pomeron factorizes into two independent pieces depending respectively on the transverse variables of the upper vertex ($E^{n,\nu}(\rho_1,\rho_2)$) and the lower vertex ($\bar{E}^{n,\nu}(\rho_1',\rho_2')$). This implies that each of the projected vertex functions $I^{n,\nu}$ and $\bar{I}^{n,\nu}$ must be invariant under the rotations in this transverse plane. On one hand, the impact factor $\phi(\rho_1,\rho_2)$ couples two external spin-1 particles. This means that in the momentum space, it writes $\tilde{\phi}_{h_1h_2}(k_1^\perp,k_2^\perp) = \epsilon^\mu_{h_1} T_{\mu\nu}(k_1^\perp,k_2^\perp)\epsilon^\nu_{h_2}$, where ϵ_{h_i} are the polarization vectors of the external photons and $T_{\mu\nu}$ is a tensor. From Lorentz-covariance and parity-conservation arguments, one finds that $T_{\mu\nu} = t_1(k_1^\perp,k_2^\perp)g_{\mu\nu} + t_2(k_1^\perp,k_2^\perp)(k^\perp_{1,\mu}k^\perp_{2,\nu} - k^\perp_{2,\mu}k^\perp_{1,\nu})$, where t_1 and t_2 are two scalar functions. This means that $\phi(\rho_1,\rho_2)$ can be written as the sum of a term transforming as a scalar, i.e. invariant under the rotations $\rho_{1,2} \to e^{i\varphi}\rho_{1,2}$, and another term transforming as a tensor, i.e. which picks up a factor $e^{\pm 2i\varphi}$ under the same rotation. On the other hand, $E^{n,\nu}(\rho_1,\rho_2)$ are the eigenfunctions of the Casimir operators of the conformal algebra, so they pick a factor $e^{in\varphi}E^{n,\nu}(\rho_1,\rho_2)$. For $I^{n,\nu}$ to be invariant, one sees that the only values for n are $|n| = 0, 2$.

This impact-factor ϕ was computed in [10] by evaluating the relevant Feynman graphs (one of these graphs is depicted in Fig. 2). Two cases were distinguished. Either the initial off-shell photon scatters into a real photon of same helicity, or the helicity undergoes a flip. It was argued that the longitudinal polarization of the virtual photon does not contribute at LLx. Let us recast the obtained expressions in the following form:

$$\begin{aligned}\phi(\rho_1,\rho_2) &= -4\pi^2\alpha_e\alpha_s e^2 \int_0^1 d\alpha f(\alpha) \int dr_1 d\bar{r}_1 \int dr_2 d\bar{r}_2 \\ &\times e^{i\alpha \mathrm{Re}(\bar{q}r_1)} e^{i(1-\alpha)\mathrm{Re}(\bar{q}r_2)} \frac{(r_1-r_2)^\delta (\bar{r}_1-\bar{r}_2)^{\tilde{\delta}}}{|r_1-r_2|^2} \\ &\times \hat{Q}K_1(|r_1-r_2|\hat{Q})\left(\delta^2(r_1-\rho_1) - \delta^2(r_2-\rho_1)\right) \\ &\times \left(\delta^2(r_1-\rho_2) - \delta^2(r_2-\rho_2)\right)\ ,\end{aligned} \tag{2.5}$$

where $\hat{Q} = \sqrt{\alpha(1-\alpha)}\,Q$, $e^2 = \sum_q e_q^2$ and $\delta = (1-\Delta)/2$, $\tilde{\delta} = (1+\Delta)/2$. The expressions for the function $f(\alpha)$ and the exponent Δ depend on the helicity. For the helicity-conserving processes $(+ \to +)$ and $(- \to -)$, $f(\alpha) = \alpha^2 + (1-\alpha)^2$ and $\Delta = 0$. In the helicity-flip case, $f(\alpha) = 2\alpha(1-\alpha)$, $\Delta = -2$ for the $(+ \to -)$ transition and $\Delta = +2$ for $(- \to +)$. The "+" and "−" refer to the helicity of the initial state (resp. final state) photon with respect to the standard basis:

$$\epsilon_\pm = \mp\frac{1}{\sqrt{2}}(1, \pm i)\ . \tag{2.6}$$

The computation of $I^{n,\nu}$ is done in the next section.

3 Projection on the conformal eigenfunctions

Let us now compute the vertex function $I^{n,\nu}$. Along the lines of [10], one inserts (2.5) into (2.3). The product of δ-functions present in the impact-factor (2.5) can be expanded. Then two of the terms correspond to the coupling of the BFKL pomeron to a single quark line and vanish when projected on the $E^{n,\nu}$. Indeed, terms of the form $\delta^2(r_1-\rho_1)\delta^2(r_2-\rho_2)$ can be rearranged to read $\delta^2(\rho_2-\rho_1)\delta^2(2r_1-\rho_1-\rho_2)$, and they give a zero contribution since $E^{n,\nu}(\rho_1,\rho_2)=0$ for $\rho_1=\rho_2$. Taking into account the symmetry $\alpha\to 1-\alpha$ of $f(\alpha)$, the two remaining contributions are identical and one obtains:

$$I^{n,\nu}=8\pi^2\alpha_e\alpha_s e^2\int_0^1 d\alpha f(\alpha)\hat{Q}\int d\rho_1 d\bar{\rho}_1\int d\rho_2 d\bar{\rho}_2 E^{n,\nu} \times(\rho_1,\rho_2)K_1(|\rho_1-\rho_2|\hat{Q}) \times e^{i\alpha\mathcal{R}\mathrm{e}(\bar{q}\rho_1)}e^{i(1-\alpha)\mathcal{R}\mathrm{e}(\bar{q}\rho_2)}\frac{(\rho_1-\rho_2)^{\delta}(\bar{\rho}_1-\bar{\rho}_2)^{\tilde{\delta}}}{|\rho_1-\rho_2|^2}\ . \tag{3.7}$$

In the following, the calculation will only be done for positive n. One inserts the expression for $E^{n,\nu}$ in (3.7) and one takes the Mellin representation of the Bessel function:

$$K_1(\mathcal{A})=\frac{1}{2}\int\frac{ds}{2i\pi}\left(\frac{\mathcal{A}}{2}\right)^{-2s-1}\Gamma(s)\Gamma(1+s)\ , \tag{3.8}$$

where the contour of integration is parallel to the z-axis and $\mathcal{R}\mathrm{e}(s)>0$. The changes of variable $\rho_1=b(1+t)$ and $\rho_2=b(1-t)$ enable to reduce one of the {holomorphic} ×{antiholomorphic} integrals in (3.7). Indeed, in these new coordinates, it is possible to factorize and perform the integration over b. This integral makes sense provided that $\gamma-\tilde{\gamma}$ is an integer. The result reads:

$$\int db\, d\bar{b}\, b^{\gamma-1}\bar{b}^{\tilde{\gamma}-1}e^{\frac{i}{2}(\mathcal{Q}\bar{b}+\bar{\mathcal{Q}}b)} = 2i\pi\ e^{i\frac{\pi}{2}(\gamma-\tilde{\gamma})}\frac{\Gamma\left(\frac{\gamma+\tilde{\gamma}}{2}+\frac{|\gamma-\tilde{\gamma}|}{2}\right)}{\Gamma\left(1-\frac{\gamma+\tilde{\gamma}}{2}+\frac{|\gamma-\tilde{\gamma}|}{2}\right)}\left(\frac{2}{\bar{\mathcal{Q}}}\right)^{\gamma}\left(\frac{2}{\mathcal{Q}}\right)^{\tilde{\gamma}} \tag{3.9}$$

$$= 2i\pi\ e^{i\frac{\pi}{2}(\gamma-\tilde{\gamma})}\frac{\Gamma(\gamma)}{\Gamma(1-\tilde{\gamma})}\left(\frac{2}{\bar{\mathcal{Q}}}\right)^{\gamma}\left(\frac{2}{\mathcal{Q}}\right)^{\tilde{\gamma}} \quad \text{if } \gamma-\tilde{\gamma}\geq 0\ , \tag{3.10}$$

with the parameter values $\mathcal{Q}=q(1-(1-2\alpha)t)$, $\gamma=1/2-a-s+\delta$, $\tilde{\gamma}=1/2-\tilde{a}-\tilde{s}+\tilde{\delta}$, and the convention $\tilde{s}=s$. Next, the integral over t can be performed. It is of the form:

$$\int dt\ t^{-\frac{3}{2}+a-s+\delta}(1-t^2)^{-a}(1-(1-2\alpha)t)^{-\frac{1}{2}+a+s-\delta} \times\{\text{a.h.}\}\ , \tag{3.11}$$

where we did not write extensively the antiholomorphic part, but it can be obtained by taking the complex conjugate of b and the "tilde" of the exponents.

The conformal mapping $t\to t/(2-t)$ leads, up to an overall factor $2^{-a-\tilde{a}+2s}$, to a well-known holomorphic integral[1]. Here again, to make sense, all the differences $a_i-\tilde{a}_i$, $b_i-\tilde{b}_i$ are integer [12]:

$$\begin{aligned}&\int dt\ t^{a_1-1}(1-t)^{b_1-a_1-1}(1-(1-\alpha)t)^{b_0-a_0-1}\times\{\text{a.h.}\}\\ &=2i\frac{\mu}{\sin\pi b_1}\frac{\pi^2}{\Gamma(b_1-a_0)\Gamma(b_1-a_1)}\left\{\frac{\Gamma(\tilde{a}_0)\Gamma(\tilde{a}_1)}{\Gamma(\tilde{b}_1-\tilde{a}_0)\Gamma(\tilde{b}_1-\tilde{a}_1)}\right.\\ &\times\frac{\alpha^{b_1-a_0-a_1}\bar{\alpha}^{\tilde{b}_1-\tilde{a}_0-\tilde{a}_1}}{\Gamma(1-a_0)\Gamma(1-a_1)}\\ &\times{}_2G_1(b_1-a_0,b_1-a_1;b_1;1-\alpha)\\ &\times{}_2G_1(\tilde{b}_1-\tilde{a}_0,\tilde{b}_1-\tilde{a}_1;\tilde{b}_1;1-\bar{\alpha})\\ &-\frac{(1-\alpha)^{1-b_1}(1-\bar{\alpha})^{1-\tilde{b}_1}}{\Gamma(1-b_1+a_0)\Gamma(1-b_1+a_1)}\\ &\times{}_2G_1(a_0-b_1+1,a_1-b_1+1;2-b_1;1-\alpha)\\ &\left.\times{}_2G_1(\tilde{a}_0-\tilde{b}_1+1,\tilde{a}_1-\tilde{b}_1+1;2-\tilde{b}_1;1-\bar{\alpha})\right\}\ ,\end{aligned} \tag{3.12}$$

where ${}_2G_1(\mathcal{A},\mathcal{B};\mathcal{C};z)\equiv\Gamma(\mathcal{A})\Gamma(\mathcal{B})/\Gamma(\mathcal{C})\times{}_2F_1(\mathcal{A},\mathcal{B};\mathcal{C};z)$, and

$$\mu=(-1)^{a_0-\tilde{a}_0}\frac{\Gamma(\tilde{b}_1-\tilde{a}_1)}{\Gamma(1-b_1+a_1)}\frac{\Gamma(1-\tilde{a}_0)}{\Gamma(a_0)}\ . \tag{3.13}$$

In our case, the parameter values are $a_0=1/2-a-s+\delta$, $a_1=-1/2+a-s+\delta$, $b_0=1$, $b_1=1/2-s+\delta$. One sees that the convergence of the integral (3.12) imposes to chose the integration contour in s such that $\mathcal{R}\mathrm{e}(s)<1/2$.

At this stage, a comment on the possible values for n is in order. Thanks to the relation $a_0+a_1=2b_1-1$, the conformal mapping $t\to -t/(t-1)$ applied to (3.12) leads to the same solution again but for a factor $(-1)^n$ and the interchange $1-\alpha\leftrightarrow\alpha$. As we integrate over α and as all the other factors depending on α are symmetric under the exchange $\alpha\leftrightarrow 1-\alpha$, we see that the final expression will have a factor $(1+(-1)^n)/2$, and thus only the even n contribute. This is expected since the BFKL pomeron has only even conformal spin-components [13]. Physically, this comes from the fact that the 4-gluon Green's function which is the BFKL kernel is symmetrized with respect to the gluons. This symmetrization corresponds to the exchange $\rho_1\leftrightarrow\rho_2$ which changes $E^{n,\nu}(\rho_1,\rho_2)$ in $(-1)^nE^{n,\nu}(\rho_1,\rho_2)$. Nevertheless, our method applies for any n and the final result will be a further check of the validity of the whole calculation.

[1] Note that our calculation differs at this level from the ones in [10,11] but after the integration over α, we obtain the same result, see (3.27) and appendix **D** for the comparison.

All in all, we arrive at the following intermediate expression (we used $\delta+\tilde{\delta}=1$):

$$
\begin{aligned}
I^{n,\nu} &= 32\pi^3(-1)^{\frac{n}{2}+\frac{\delta-\tilde{\delta}}{2}+\frac{1}{2}+\delta}\alpha_e\alpha_s e^2\left(\frac{Q}{2}\right)^{a+\tilde{a}-2} \\
&\times \sin\pi a\frac{\Gamma(1-\tilde{a})}{\Gamma(a)}\int\frac{ds}{2i\pi}\frac{\Gamma(s)\Gamma(1+s)}{\sin\pi s} \\
&\times \frac{1}{\Gamma(\frac{3}{2}-\delta-a+s)\Gamma(\frac{1}{2}-\delta+a+s)}\left(\frac{q}{Q}\right)^{-\frac{1}{2}+\tilde{a}+s-\tilde{\delta}} \\
&\times \left(\frac{\bar{q}}{Q}\right)^{-\frac{1}{2}+a+s-\delta}\Bigg\{\Gamma(-\frac{1}{2}+\tilde{\delta}+\tilde{a}-s) \\
&\times\Gamma(\frac{1}{2}+\tilde{\delta}-\tilde{a}-s)\sin\pi\tilde{a} \\
&\times\int_0^1 d\alpha\ f(\alpha)\alpha^s(1-\alpha)^{-s}\ {}_2G_1(a,1-a;\frac{1}{2}-s+\delta;1-\alpha) \\
&\times{}_2G_1(\tilde{a},1-\tilde{a};\frac{1}{2}-s+\tilde{\delta};1-\alpha) \\
&-\Gamma(\frac{3}{2}-\delta-a+s)\Gamma(\frac{1}{2}-\delta+a+s) \\
&\times\sin\pi a\int_0^1 d\alpha\ f(\alpha)\alpha^{-s}(1-\alpha)^s \\
&\times{}_2G_1(a,1-a;\frac{3}{2}+s-\delta;1-\alpha) \\
&\times{}_2G_1(\tilde{a},1-\tilde{a};\frac{3}{2}+s-\tilde{\delta};1-\alpha)\Bigg\}\ . \qquad (3.14)
\end{aligned}
$$

The integrations over α remain to be performed. We can treat all the cases by computing the following generic integral:

$$
\begin{aligned}
\mathcal{J}^m(a,c) &= \frac{\sin\pi a}{\pi}\frac{\sin\pi\tilde{c}}{\pi}\Gamma(\tilde{c}-\tilde{a})\Gamma(\tilde{c}+\tilde{a}-1) \\
&\times\int_0^1 d\alpha\ \alpha^{m+1-c}(1-\alpha)^{c-1} \\
&\times{}_2G_1(a,1-a;c;1-\alpha) \\
&\times{}_2G_1(\tilde{a},1-\tilde{a};\tilde{c};1-\alpha)\ , \qquad (3.15)
\end{aligned}
$$

where $c=1/2-s+\delta$ for the first integral and $c=3/2+s-\delta$ for the second one. For simplicity, we consider that when one of the arguments of $\mathcal{J}$ has a "tilde" it means that we exchange in the formula the corresponding argument with its "tilde" counterpart. This notation can be slightly misleading since the "tilde"-operation is not involutive. The index $m\in\{0,1,2\}$ has been introduced in order to take into account the functions $f(\alpha)$ which we rewrite $1-2\alpha+2\alpha^2$ and $2\alpha(1-\alpha)$ respectively. Thus the "physical" integrals to compute are:

$$
\begin{aligned}
\mathcal{J}_{++}(a,c) &= \mathcal{J}^0(a,c)-2\mathcal{J}^1(a,c)+2\mathcal{J}^2(a,c) \\
\mathcal{J}_{+-}(a,c) &= 2\mathcal{J}^2(a,c)\ . \qquad (3.16)
\end{aligned}
$$

With these notations, the amplitudes can be written:

$$
\begin{aligned}
I^{n,\nu}_{h_1h_2} &= 32\pi^4(-1)^{\frac{n}{2}+\frac{\delta-\tilde{\delta}}{2}+\frac{1}{2}+\delta}\alpha_e\alpha_s e^2\left(\frac{Q}{2}\right)^{a+\tilde{a}-2} \\
&\times\sin\pi a\frac{\Gamma(1-\tilde{a})}{\Gamma(a)}\int\frac{ds}{2i\pi}\frac{\Gamma(s)\Gamma(1+s)}{\sin\pi s} \\
&\times\frac{1}{\Gamma(\frac{3}{2}-\delta-a+s)\Gamma(\frac{1}{2}-\delta+a+s)}\left(\frac{q}{Q}\right)^{-\frac{1}{2}-\tilde{\delta}+\tilde{a}+s} \\
&\times\left(\frac{\bar{q}}{Q}\right)^{-\frac{1}{2}-\delta+a+s}(-1)^n \\
&\times\Bigg\{\frac{\pi}{\sin\pi(\frac{1}{2}+\tilde{\delta}-s)}\mathcal{J}_{h_1h_2}(a,\frac{1}{2}+\delta-s) \\
&-\frac{\pi}{\sin\pi(\frac{3}{2}-\delta+s)}\mathcal{J}_{h_1h_2}(1-\tilde{a},\frac{3}{2}-\tilde{\delta}+s)\Bigg\}\ , \qquad (3.17)
\end{aligned}
$$

where $h_1,h_2\in\{+,-\}$.

We can directly integrate (3.15). The method is to replace one of the ${}_2G_1(1-\alpha)$ in formula (3.15) by a sum of ${}_2G_1(\alpha)$. A consequence is that the result is invariant under the exchange $a\leftrightarrow\tilde{a}$, because of the 2 possible choices for doing this replacement. It leads to ${}_4G_3$ functions (see appendix **E**), for which the explicit cancellation for $|n|>2$ appears. But we have a method which leads directly to the ${}_3G_2$ functions that we explicitly compute in the appendix. We express ${}_2G_1(z)$ as a $G^{22}_{22}(1-z)$ Meijer function, namely:

$$
\begin{aligned}
{}_2G_1(\mathcal{A},\mathcal{B};\mathcal{C};z) &= \frac{1}{\Gamma(\mathcal{C}-\mathcal{A})\Gamma(\mathcal{C}-\mathcal{B})}G^{22}_{22} \\
&\times\left(\begin{matrix}1-\mathcal{A},1-\mathcal{B}\\ \mathcal{C}-A-B\end{matrix};1-z\right) \\
&= \frac{1}{\Gamma(\mathcal{C}-\mathcal{A})\Gamma(\mathcal{C}-\mathcal{B})}\int\frac{ds'}{2i\pi}(1-z)^{s'} \\
&\times\Gamma(-s')\ \Gamma(\mathcal{C}-\mathcal{A}-\mathcal{B}-s') \\
&\times\Gamma(\mathcal{A}+s')\Gamma(\mathcal{B}+s')\ . \qquad (3.18)
\end{aligned}
$$

Inserting this identity into (3.15) then performing the integration over α [14], one obtains:

$$
\begin{aligned}
\mathcal{J}^m(a,c) &= \frac{\sin\pi\tilde{c}}{\pi}\int\frac{ds'}{2i\pi}\Gamma(\tilde{a}+s')\Gamma(1-\tilde{a}+s') \qquad (3.19) \\
&\times\left\{\frac{\Gamma(-s')\Gamma(m+1+s')\Gamma(\tilde{c}-1-s')\Gamma(m+2-c+s')}{\Gamma(m+2-a+s')\Gamma(m+1+a+s')}\right\}\ .
\end{aligned}
$$

Next, we transform this integral into a sum of ${}_3G_2$ functions, which is done by reducing the quotient of Γ-functions between the brackets in the r.h.s. and then constructing the defining series for the ${}_3G_2$ by picking the poles. This goes as follows. One writes:

$$\left\{\dots\right\} = \frac{\pi}{\sin\pi(-s')}\frac{\pi}{\sin\pi(\tilde{c}-1-s')}$$
$$\times\left[\frac{\prod_{j=0}^{m-1}(1+s'+j)\ \times\ \Gamma(m+2-c+s')/\Gamma(2-\tilde{c}+s')}{\prod_{j=0}^{k-1}(m+1+a+s'-k+j)(m+2-a+s'-k+j)}\right]$$
$$\times\frac{1}{\Gamma(m+1+a+s'-k)\Gamma(m+2-a+s'-k)}\ . \qquad (3.20)$$

By simple inspection, we get for any m:

$$\left\{\dots\right\} = \frac{\pi}{\sin\pi(-s')}\frac{\pi}{\sin\pi(\tilde{c}-1-s')}$$
$$\times\sum_{p,q=0}^{m}\frac{\mathcal{A}_{pq}(a,c)}{\Gamma(1+a+p+s')\Gamma(2-a+q+s')}\ , \qquad (3.21)$$

where the $\mathcal{A}_{pq}$s do not depend on s'. The integral defining $\mathcal{J}$ can then be computed by constructing two series whose coefficients are the residues at the right poles of the two inverse-sines. Each of the series is a ${}_3G_2$-function of the type of those computed in appendix **A**. The values for the non-vanishing coefficients $\mathcal{A}_{pq}$ we have chosen are:

$$\mathcal{A}_{00}=1,\ \mathcal{A}_{01}=\frac{2}{2a-1}\left(a(a-1)+c(2-a)+c^2\right),$$
$$\mathcal{A}_{10}=\frac{2}{1-2a}\left(a(a-1)+c(a+1)+c^2\right),$$
$$\mathcal{A}_{12}=\frac{2}{2a-1}(a-1)(a-2)(a-c)(a-c-1),$$
$$\mathcal{A}_{21}=\frac{2}{1-2a}a(a+1)(a+c)(a+c-1) \qquad (3.22)$$

for the helicity-conserving $(+\to+)$ amplitude and

$$\mathcal{A}_{11}=2,\ \mathcal{A}_{12}=\frac{2}{2a-1}(a(a-3)+2),\ \mathcal{A}_{21}=\frac{2}{1-2a}a(a+1) \qquad (3.23)$$

for the helicity-flip $(+\to-)$ one. Let us complete the calculation for the $\mathcal{J}$'s.

$$\mathcal{J}_{h_1h_2}(a,c)=\sum_{p,q}\mathcal{A}_{h_1h_2,pq}(a,c)$$
$$\times\left\{{}_3G_2\left(\begin{matrix}1,\tilde{c}+\tilde{a}-1,\tilde{c}-\tilde{a}\\ \tilde{c}+a+p,\tilde{c}-a+1+q\end{matrix};1\right)\right.$$
$$\left.-{}_3G_2\left(\begin{matrix}1,\tilde{a},1-\tilde{a}\\ 1+a+p,2-a+q\end{matrix};1\right)\right\}\ , \qquad (3.24)$$

where we have been able to factorize and simplify $\pi/\sin\pi\tilde{c}$ since c and $\tilde{c}$ differ by an even integer (0 or 2) in all cases. The differences of ${}_3G_2$ in the r.h.s. can be computed using the tricks exposed in appendix **A** and **B**. The ones needed for our purpose are explicitly shown in appendix **C**.

However, formula (A.40) enables us to perform the calculation for general n (and then take the appropriate limits). It enables us to write the $\mathcal{J}$s as a sum of rational fractions in a, $\tilde{a}$ and c, times a non-rational coefficient which is either $\sin\pi a/\sin\pi\tilde{a}$ or $\Gamma(c-\tilde{a})\Gamma(c+\tilde{a}-1)/\Gamma(c-a)\Gamma(c+a-1)$. The rational fractions are tedious but straightforward to compute using `mathematica`. Inserting (3.24) after appropriate computation into (3.17), and recalling that $a=(1-n)/2+i\nu$ yield for $n=0$:

$$\mathcal{J}_{++}(\tfrac{1}{2}+i\nu,1-s)+\mathcal{J}_{++}(\tfrac{1}{2}-i\nu,1+s)$$
$$=\frac{\pi}{16i\nu(1+\nu^2)}\frac{\sin^2\pi s\tan\pi i\nu}{\cos\pi(i\nu-s)\cos\pi(i\nu+s)}(11+12\nu^2+4s^2)$$
$$\mathcal{J}_{+-}(\tfrac{1}{2}+i\nu,2-s)+\mathcal{J}_{+-}(\tfrac{1}{2}-i\nu,2+s)$$
$$=\frac{\pi}{4i\nu(1+\nu^2)}\frac{\sin^2\pi s\tan\pi i\nu}{\cos\pi(i\nu-s)\cos\pi(i\nu+s)}\ . \qquad (3.25)$$

Those corresponding to $n=2$ read:

$$\mathcal{J}_{++}(-\tfrac{1}{2}+i\nu,1-s)+\mathcal{J}_{++}(-\tfrac{1}{2}-i\nu,1+s)$$
$$=-\frac{\pi}{32i\nu(1+\nu^2)}\frac{\sin^2\pi s\tan\pi i\nu}{\cos\pi(i\nu-s)\cos\pi(i\nu+s)}$$
$$\times(-1+2i\nu-2s)(1+2i\nu-2s)$$
$$\mathcal{J}_{+-}(-\tfrac{1}{2}+i\nu,2-s)+\mathcal{J}_{+-}(-\tfrac{1}{2}-i\nu,2+s)$$
$$=-\frac{\pi}{8i\nu(1+\nu^2)}\frac{\sin^2\pi s\tan\pi i\nu}{\cos\pi(i\nu-s)\cos\pi(i\nu+s)}\ . \qquad (3.26)$$

We find that all the other components are zero.

Let us summarize our final results.

We obtain the selection rule that for a virtual photon scattering, only the components $n=0$ and $n=\pm2$ contribute. We list underneath the expressions for these non-vanishing amplitudes. We introduce the angle ϕ between the plane formed by the initial- and final-state electrons and the initial- and final-state photons respectively, which is the argument of q. First, the $n=0$ components:

$$I^{0,\nu}_{++}=-2\pi^6\alpha_e\alpha_s e^2\left(\frac{Q}{2}\right)^{-1+2i\nu}\frac{\tanh\pi\nu}{\pi\nu(\nu^2+1)}\frac{1}{\Gamma^2(\frac{1}{2}+i\nu)}$$
$$\times\int\frac{ds}{2i\pi}\left(\frac{q^2}{Q^2}\right)^{-\frac{1}{2}+i\nu+s}$$
$$\times\Gamma(s)\ \Gamma(1+s)\ \Gamma(\tfrac{1}{2}-i\nu-s)\ \Gamma(\tfrac{1}{2}+i\nu-s)$$
$$\times\left(11+12\nu^2+4s^2\right)$$

$$I^{0,\nu}_{+-}=8\pi^6\alpha_e\alpha_s e^2\left(\frac{Q}{2}\right)^{-1+2i\nu}\frac{\tanh\pi\nu}{\pi\nu(\nu^2+1)}\frac{1}{\Gamma^2(\frac{1}{2}+i\nu)}e^{2i\phi}$$
$$\times\int\frac{ds}{2i\pi}\left(\frac{q^2}{Q^2}\right)^{-\frac{1}{2}+i\nu+s}$$
$$\times\Gamma(s)\ \Gamma(1+s)\ \Gamma(\tfrac{3}{2}-i\nu-s)\ \Gamma(\tfrac{3}{2}+i\nu-s)\ . \qquad (3.27)$$

We checked that these two expressions agree with [10] but for an overall sign difference.

Second, the $n=2$ components:

$$I^{2,\nu}_{++} = -4\pi^6 \alpha_e \alpha_s e^2 \left(\frac{Q}{2}\right)^{-1+2i\nu} \frac{\tanh \pi\nu}{\pi\nu(\nu^2+1)} \frac{1}{\Gamma^2(\frac{1}{2}+i\nu)} \times \frac{-\frac{1}{2}+i\nu}{+\frac{1}{2}+i\nu} e^{2i\phi} \times \int \frac{ds}{2i\pi} \left(\frac{q^2}{Q^2}\right)^{-\frac{1}{2}+i\nu+s} \times \Gamma(s)\,\Gamma(1+s)\,\Gamma(\frac{3}{2}-i\nu-s)\,\Gamma(\frac{3}{2}+i\nu-s)$$

$$I^{2,\nu}_{+-} = 4\pi^6 \alpha_e \alpha_s e^2 \left(\frac{Q}{2}\right)^{-1+2i\nu} \frac{\tanh \pi\nu}{\pi\nu(\nu^2+1)} \frac{1}{\Gamma^2(\frac{1}{2}+i\nu)} \times \frac{-\frac{1}{2}+i\nu}{+\frac{1}{2}+i\nu} e^{4i\phi} \times \int \frac{ds}{2i\pi} \left(\frac{q^2}{Q^2}\right)^{-\frac{1}{2}+i\nu+s} \times \Gamma(s)\,\Gamma(1+s)\,\Gamma(\frac{1}{2}+i\nu-s)\,\Gamma(\frac{5}{2}-i\nu-s). \quad (3.28)$$

Third, the $n=-2$ components are deduced from the preceeding ones using an appropriate relation between $E^{n,\nu}$ and $E^{-n,\nu}$. Let us briefly derive this relation. We come back to (3.7) and note that it factorizes in the following way by performing the change of variable $\rho_1 = b+\rho/2$ and $\rho_2 = b-\rho/2$:

$$I^{n,\nu} = \int_0^1 d\alpha \int d\rho\, d\bar\rho\; \mathcal{H}(\alpha,\rho)\, \pi^3 \frac{2^{4i\nu}}{-i\nu+n/2} \times \frac{\Gamma(-i\nu+(1+n)/2)}{\Gamma(i\nu+(1+n)/2)} \frac{\Gamma(i\nu+n/2)}{\Gamma(-i\nu+n/2)} E^{n,\nu}_q(\rho)\ , \quad (3.29)$$

where $E^{n,\nu}_q(\rho)$ is defined in [7]. We put in the function $\mathcal{H}(\alpha,\rho)$ all the other dependencies, namely:

$$\mathcal{H}(\alpha,\rho) = 8i\,\alpha_e\alpha_s\, e^2\, f(\alpha)\hat{Q}\, e^{i(\alpha-1/2)\mathcal{R}\mathrm{e}(\bar q\rho)} \times K_1(|\rho|\hat{Q}) \frac{\rho^{\delta}\bar\rho^{\bar\delta}}{|\rho|}\ . \quad (3.30)$$

Note that this formula shows that $I^{n,\nu}$ is the projection of the impact factor on $E^{n,\nu}_q$ [15]. We then notice that for positive n [7]:

$$E^{-n,\nu}_q(b) = 2^{-12i\nu} \frac{n/2-i\nu}{n/2+i\nu} \frac{\Gamma^2(n/2-i\nu)}{\Gamma^2(n/2+i\nu)} \times q^{2a-1}\bar q^{2\tilde a-1} E^{n,-\nu}_q(b)\ , \quad (3.31)$$

and hence we arrive at the relation:

$$I^{-n,\nu} = 2^{-4i\nu} \frac{\Gamma^2(-i\nu+(1+n)/2)}{\Gamma^2(i\nu+(1+n)/2)} \times q^{2a-1}\bar q^{2\tilde a-1}\; I^{n,-\nu}\ . \quad (3.32)$$

Applying this relation to (3.28), one obtains:

$$I^{-2,\nu}_{++} = -4\pi^6 \alpha_e \alpha_s e^2 \left(\frac{Q}{2}\right)^{-1+2i\nu} \frac{\tanh \pi\nu}{\pi\nu(\nu^2+1)} \frac{1}{\Gamma^2(\frac{1}{2}+i\nu)} \times \frac{-\frac{1}{2}+i\nu}{+\frac{1}{2}+i\nu} e^{-2i\phi} \times \int \frac{ds}{2i\pi} \left(\frac{q^2}{Q^2}\right)^{-\frac{1}{2}+i\nu+s} \times \Gamma(s)\,\Gamma(1+s)\,\Gamma(\frac{3}{2}-i\nu-s)\,\Gamma(\frac{3}{2}+i\nu-s)$$

$$I^{-2,\nu}_{+-} = 4\pi^6 \alpha_e \alpha_s e^2 \left(\frac{Q}{2}\right)^{-1+2i\nu} \frac{\tanh \pi\nu}{\pi\nu(\nu^2+1)} \frac{1}{\Gamma^2(\frac{1}{2}+i\nu)} \times \frac{-\frac{1}{2}+i\nu}{+\frac{1}{2}+i\nu} \times \int \frac{ds}{2i\pi} \left(\frac{q^2}{Q^2}\right)^{-\frac{1}{2}+i\nu+s} \times \Gamma(s)\,\Gamma(1+s)\,\Gamma(\frac{1}{2}-i\nu-s)\,\Gamma(\frac{5}{2}+i\nu-s)\ . \quad (3.33)$$

Note that the complex integrals over s can be expressed with (one or a sum of) Legendre functions. Note also that the helicity-flip component for $n=2$ does not vanish at small q.

The other helicity amplitudes are simply obtained using the relations $I^{n,\nu}_{--} = I^{-n,\nu}_{++}|_{\phi\to-\phi}$ and $I^{n,\nu}_{-+} = I^{-n,\nu}_{+-}|_{\phi\to-\phi}$.

4 Conclusion

We have computed the coupling of a $\gamma^*\gamma$ impact-factor to the LLx BFKL pomeron. We have found that our calculation is consistent with a previous one [10] for the $n=0$ component. For the higher-conformal spin components, only the two values $n=\pm2$ contribute, as expected from symmetry considerations.

The physical motivation underneath was the precise study of diffractive photon production in the high-energy regime, including the "higher-twist" type components induced by conformal invariance [8]. With the results we obtained in this paper, we are almost ready to study the phenomenology of the higher-spin components of the BFKL pomeron with a realistic impact-factor. It should also be worth to transpose the methods developped here to the computation of other impact factors. We leave these studies for forthcoming papers.

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A A useful formula involving generalized hypergeometric functions $_3F_2$

In this appendix, one finds an appropriate summation for the hypergeometric function

$${}_3F_2\left(\begin{matrix}1, b+n, c-n\\ 1+b+p, 1+c+q\end{matrix}; 1\right)$$

for any integer n and nonnegative integer values of p and q. Note that we can stick to nonnegative n and obtain $-n$ by interchanging b and c and p and q respectively. The results are given in (A.35,A.38) or (A.40).

The existence of the $_3F_2$ function is ensured by the strict positivity of the real part of the quantity

$$s \equiv (1+b+p)+(1+c+q)-1-(b+n)-(c-n) = 1+p+q\ .$$

For our purpose, the relevant values for the parameters of the ${}_3F_2$-function are any nonnegative integer n and $p, q \in \{0,1,2\}$. An elementary method consists in expressing the function as a series of quotients of Γ functions, which reduces to a series of rational fractions due to the particular values of its arguments. The latter are decomposed as a series of terms with minimal denominators (which are first order polynomials in the summation index). One can resum the series and one finally obtains a finite sum whose terms are expressible as ratios of Γ functions and possibly (depending on the relative values of p and n) Ψ functions. However, the number of contributing terms apparently grows as n, and one cannot easily get by this method a simple expression for general n and small p and q. This remark can be illustrated by applying a transformation formula (see [16], formula (1) page 533):

$$
\begin{aligned}
&{}_3F_2\left(\begin{matrix}1, b+n, c-n\\ 1+b+p, 1+c+q\end{matrix}; 1\right)\\
&= \frac{\Gamma(1+p+q)}{\Gamma(1+n+q)}\frac{\Gamma(1+c+q)}{\Gamma(1+c+p+q-n)}\\
&\times {}_3F_2\left(\begin{matrix}p-n, b+p, c-n\\ 1+c+p+q-n, 1+b+p\end{matrix}; 1\right) \,. \qquad \text{(A.34)}
\end{aligned}
$$

One sees that in the case $n < p$, the r.h.s. of the preceeding equation is an hypergeometric polynomial which has $n - p$ terms. Thus a more sophisticated method has to be developped.

Let us distinguish the cases **(i)** $n > p$ and **(ii)** $n \leq p$.

(i) $n > p$.
This case can be obtained by immediate application of formula (6) (*Ibidem*, page 534). The condition of applicability is $\mathcal{R}\mathrm{e}(c+q) > 0$ which is satisfied in the case of interest in the core of the paper. The result reads:

$$
\begin{aligned}
&{}_3F_2\left(\begin{matrix}1, b+n, c-n\\ 1+b+p, 1+c+q\end{matrix}; 1\right)\\
&= \frac{\Gamma(1+b+p)}{\Gamma(b+n)}\frac{\Gamma(1+c+q)}{\Gamma(c-n)}\\
&\times \frac{\Gamma(c-b-n-p)}{\Gamma(1+c-b+q-n)} B(1+p+q, n-p)\\
&- \frac{(b+p)(c+q)}{(n-p)(c-b-n-p)}\\
&\times {}_3F_2\left(\begin{matrix}-p-q, 1-b-p, 1\\ 1+n-p, 1+c-b-n-p\end{matrix}; 1\right) \qquad \text{(A.35)}
\end{aligned}
$$

Note that the ${}_3F_2$ in the r.h.s. is a finite sum containing $p+q$ terms, thanks to the parameter $-p-q$.

(ii) $n \leq p$.
This case is a bit more tricky. One uses formula (3) (see [16]), but one first needs to regularize by introducing a small parameter ϵ such that:

$$
\begin{aligned}
&{}_3F_2\left(\begin{matrix}1, b+n, c-n\\ 1+b+p, 1+c+q\end{matrix}; 1\right)\\
&= \lim_{\epsilon\to 0} {}_3F_2\left(\begin{matrix}1+\epsilon, b+n, c-n\\ 1+b+p, 1+c+q\end{matrix}; 1\right) \,.
\end{aligned}
$$

This leads to:

$$
\begin{aligned}
&{}_3F_2\left(\begin{matrix}1+\epsilon, b+n, c-n\\ 1+b+p, 1+c+q\end{matrix}; 1\right) = \Gamma(1+b+p)\Gamma(1+c+q)\\
&\times \Gamma(-\epsilon)\left\{\frac{\Gamma(c-b-2n)}{\Gamma}\right.\\
&\times (1+p-n)\Gamma(1+c-b+q-n)\Gamma(b+n-\epsilon) \qquad \text{(A.36)}\\
&\left.\times \sum_{k=0}^{\infty} \frac{(b+n)_k (b-c+n-q)_k (n-p)_k}{\Gamma(1+k)(b+n-\epsilon)_k (1+b-c+2n)_k} + \begin{bmatrix} b \leftrightarrow c\\ n \leftrightarrow -n\\ p \leftrightarrow q\end{bmatrix}\right\},
\end{aligned}
$$

where we have employed the classical notation $(\mathcal{A})_k = \Gamma(\mathcal{A}+k)/\Gamma(\mathcal{A})$, and the square brackets is a short notation for the term obtained by the indicated exchange of parameters. The next step is to take the limit $\epsilon \to 0$. One uses the fact that $\Gamma(\mathcal{A}+\epsilon) = \Gamma(\mathcal{A})(1+\epsilon\Psi(\mathcal{A})) + \mathrm{o}(\epsilon)$ to obtain:

$$
\begin{aligned}
&{}_3F_2\left(\begin{matrix}1, b+n, c-n\\ 1+b+p, 1+c+q\end{matrix}; 1\right)\\
&= -\frac{\Gamma(1+b+p)}{\Gamma(b+n)}\frac{\Gamma(1+c+q)}{\Gamma(c-n)}\\
&\times \left\{\frac{\Gamma(c-b-2n)}{\Gamma(1+p-n)\Gamma(1+c-b+q-n)}\right.\\
&\times \sum_{k=0}^{p-n} \frac{(b-c-q+n)_k (n-p)_k}{\Gamma(1+k)(1+b-c+2n)_k} \left(\Psi(b+n+k) - \Psi(b+n)\right)\\
&+ \frac{\Gamma(c-b-2n)}{\Gamma(1+p-n)\Gamma(1+c-b+q-n)}\frac{\Gamma(1+p+q)}{\Gamma(1+n+q)}\\
&\left.\times \frac{\Gamma(1+b-c+2n)}{\Gamma(1+b-c+n+p)}\Psi(b+n) + \begin{bmatrix} b \leftrightarrow c\\ n \leftrightarrow -n\\ p \leftrightarrow q\end{bmatrix}\right\} \,. \qquad \text{(A.37)}
\end{aligned}
$$

We wrote it in this form in order to isolate the special functions by using the summation formula (B.44) in appendix **B**. After a bunch of straightforward manipulations, we are led to the following result:

$$
\begin{aligned}
&{}_3F_2\left(\begin{matrix}1, b+n, c-n\\ 1+b+p, 1+c+q\end{matrix};1\right)\\
&= -\frac{\Gamma(1+b+p)}{\Gamma(b+n)}\frac{\Gamma(1+c+q)}{\Gamma(c-n)}\Bigg\{(-1)^{p-n}\binom{p+q}{p-n}\\
&\times\frac{\Gamma(c-b-n-p)}{\Gamma(1+c-b+q-n)}\left(\Psi(b+n)-\Psi(c-n)\right)\\
&+\Bigg[\frac{\pi}{\sin\pi b}\frac{\Gamma(c-b-p-n)\Gamma(b+n)}{\Gamma(1+q+n)}\\
&\times\sum_{k=0}^{p-n-1}\frac{1}{n-p+k}\frac{1}{\Gamma(1+k)}\frac{\Gamma(1+q+n+k)\Gamma(1-b-p+k)}{\Gamma(1+c-b+q-p+k)}\\
&+\left[\begin{matrix}b\leftrightarrow c\\ n\leftrightarrow -n\\ p\leftrightarrow q\end{matrix}\right]\Bigg]\Bigg\}\ . \qquad \text{(A.38)}
\end{aligned}
$$

The result has been put in a form which shows that if $c = 1-b$, as it is the case in the core of this paper, the difference of Ψ reduces to a tangent, through the identity:

$$\Psi(1-x)-\Psi(x) = \frac{\pi}{\tan\pi x}\ . \qquad \text{(A.39)}$$

One of the most interesting points in these formulae is that the number of terms is at most equal to $p+q$, regardless the value of n, making them particularly adapted for our purpose.

Note that in the two cases $n = 0$ and $\{p, q\} = 0, 1$, the obtained formula matches with formerly calculated expressions, see for instance [16].

We can obtain a plethora of such formulae by using the relations between the various ${}_3F_2$ functions. A particularly interesting one is the following:

$$
\begin{aligned}
&{}_3F_2\left(\begin{matrix}1, b+n, c-n\\ 1+b+p, 1+c+q\end{matrix};1\right)\\
&= \frac{\Gamma(1+b+p)\Gamma(1+c+q)\Gamma(c-b-n-p)}{\Gamma(b+n)\Gamma(c-n)\Gamma(1+c-b-n+q)}\\
&\times\sum_{i=0}^{p+q}\frac{(-1)^i}{n-p+i}\binom{p+q}{i}\\
&\times\left(1-\frac{\Gamma(b+n+i)}{\Gamma(b+p)}\frac{\Gamma(c-n)}{\Gamma(c-p+i)}\right)\ . \qquad \text{(A.40)}
\end{aligned}
$$

(We do not reproduce the proof here, but for $p < n$, it mainly relies on the use of formula (25) on page 108 in [17]). Note that the formula is trivially analytical for $n > p$, but also in the limit $n \equiv m \leq p$: all the terms are analytical but for one pole appearing as a denominator $1/(n-p+i)$ for the appropriate value of i. However, its residue vanishes and so the expression in the r.h.s is finite. We checked numerically that the obtained expression is correct for the values of p, q of interest. The property is not trivial since the asymptotic behaviour of the given ${}_3F_2$ does not satisfy the hypotheses for Carlson's theorem and hence there are infinitely many ways to continue analytically $n > p$.

B A summation formula

For the needs of appendix **A**, we simplify the following expression, getting rid of the Ψ-functions:

$$\sum_{k=0}^{l}\frac{1}{\Gamma(1+k)}\frac{(-l)_k(\alpha)_k}{(\gamma)_k}\left\{\Psi(\beta+k)-\Psi(\beta)\right\}\ .$$

One notes that:

$$\Psi(\beta+k)-\Psi(\beta) = \sum_{i=0}^{k-1}\frac{1}{\beta+i}\ ,$$

and one defines the function:

$$f(z) = \sum_{k=0}^{l}\frac{1}{\Gamma(1+k)}\frac{(-l)_k(\alpha)_k}{(\gamma)_k}\left\{\sum_{i=0}^{k-1}\frac{z^{\beta+i}}{\beta+i}\right\}\ , \qquad \text{(B.41)}$$

z being an arbitrary parameter. The quantity we need to compute is $f(1)$. This goes as follows. The derivative of f can be summed and we obtain the difference of two hypergeometric ${}_2F_1$ functions:

$$f'(z) = \frac{z^{\beta-1}}{z-1}\left\{{}_2F_1(-l,\alpha,\gamma,z)-{}_2F_1(-l,\alpha,\gamma,1)\right\}\ . \qquad \text{(B.42)}$$

One uses a well-known transformation [14] for the first term in the r.h.s. of the preceeding equation:

$$
\begin{aligned}
{}_2F_1(-l,\alpha,\gamma,z) &= (1-z)^l\frac{\Gamma(\gamma)\Gamma(\alpha+l)}{\Gamma(\alpha)\Gamma(\gamma+l)}\\
&\times{}_2F_1(-l,\gamma-\alpha,1-l-\alpha,1/(1-z))\ . \qquad \text{(B.43)}
\end{aligned}
$$

The integration over z can then be performed safely for $\mathcal{R}\mathrm{e}(\beta) > 0$. After a few easy manipulations, the result can be written in the following compact form:

$$
\begin{aligned}
&\sum_{k=0}^{l}\frac{1}{\Gamma(1+k)}\frac{(-l)_k(\alpha)_k}{(\gamma)_k}\left\{\Psi(\beta+k)-\Psi(\beta)\right\}\\
&= \frac{(\alpha)_l}{(\beta)_l(\gamma)_l}\sum_{k=0}^{l-1}\frac{1}{k-l}\frac{\Gamma(1+l)}{\Gamma(1+k)}\frac{(\gamma-\alpha)_k(1-\beta-l)_k}{(1-\alpha-l)_k}\ . \qquad \text{(B.44)}
\end{aligned}
$$

Note the following particular case occuring when $\alpha \equiv \gamma$:

$$\sum_{k=0}^{l}(-1)^{k+1}\binom{l}{k}\Psi(\beta+k) = B(\beta, l)\ . \qquad \text{(B.45)}$$

C Some particular cases

In this appendix, we compute the difference of ${}_3G_2$-functions in (3.24), using formulae (A.35,A.38) or alternatively formula (A.40). To fix the notation:

$$\mathcal{G}_{pq}(n) \equiv {}_3G_2\left(\begin{matrix}1, \gamma+\alpha-1+n, \gamma-\alpha-n \\ \gamma+\alpha+p, \gamma-\alpha+1+q\end{matrix}; 1\right) - {}_3G_2\left(\begin{matrix}1, \alpha+n, 1-\alpha-n \\ 1+\alpha+p, 2-\alpha+q\end{matrix}; 1\right) . \quad \text{(C.46)}$$

We recall that ${}_3G_2$ is related to ${}_3F_2$ through the relation:

$${}_3G_2\left(\begin{matrix}a_1, a_2, a_3 \\ b_1, b_2\end{matrix}; z\right) \equiv \frac{\Gamma(a_1)\Gamma(a_2)\Gamma(a_3)}{\Gamma(b_1)\Gamma(b_2)} \times {}_3F_2\left(\begin{matrix}a_1, a_2, a_3 \\ b_1, b_2\end{matrix}; z\right) . \quad \text{(C.47)}$$

A few of the following expressions can be obtained using [16]. For the others, we use the quoted formulae, with the parameter values $a \equiv \gamma+\alpha-1$ and $b \equiv \gamma-\alpha$ (resp. $a \equiv \alpha$ and $b \equiv 1-\alpha$). We only display the result for $n = 0$ and $n = 2$ although a general expression can be written.

$$\mathcal{G}_{00}(n=0) = \frac{1}{2\alpha-1}\left\{\pi \cot\pi\alpha + \Psi(\gamma+\alpha-1) - \Psi(\gamma-\alpha)\right\}$$

$$\mathcal{G}_{00}(n=2) = \frac{\gamma-1}{(\gamma-\alpha-1)(\gamma-\alpha-2)}$$

$$\mathcal{G}_{01}(n=0) = \frac{(\gamma-1)}{2\,(\alpha-1)^2(\gamma-\alpha)} - \frac{\pi\cot\pi\alpha + \Psi(\gamma+\alpha-1) - \Psi(\gamma-\alpha)}{2\,(\alpha-1)(2\alpha-1)}$$

$$\mathcal{G}_{01}(n=2) = \frac{(\gamma-1)(6\alpha^2-3\alpha\gamma+\gamma(\gamma-2))}{6\alpha(1-\alpha)(\alpha-\gamma)(1+\alpha-\gamma)(2+\alpha-\gamma)}$$

$$\mathcal{G}_{10}(n=0) = \frac{\gamma-1}{2\alpha^2(\gamma+\alpha-1)} + \frac{\pi\cot\pi\alpha + \Psi(\gamma+\alpha-1) - \Psi(\gamma-\alpha)}{2\alpha(2\alpha-1)}$$

$$\mathcal{G}_{10}(n=2) = \frac{1}{2}\left\{\frac{1}{(\gamma-\alpha-1)(\gamma-\alpha-2)} - \frac{1}{\alpha(1+\alpha)}\right\}$$

$$\mathcal{G}_{11}(n=0) = \frac{\frac{1}{\alpha-1} + \frac{1}{\alpha} + \frac{\alpha}{\gamma-\alpha} + \frac{\alpha-1}{\gamma+\alpha-1} - \pi\cot\pi\alpha + \Psi(\gamma-\alpha) - \Psi(\gamma+\alpha)}{2\alpha(\alpha-1)(2\alpha-1)}$$

$$\mathcal{G}_{11}(n=2) = -\frac{1}{6}\left(\frac{2}{\alpha(1-\alpha^2)} + \frac{1}{\alpha-\gamma} - \frac{2}{1+\alpha-\gamma} + \frac{1}{2+\alpha-\gamma}\right)$$

$$\mathcal{G}_{12}(n=0) = -\frac{1}{8\alpha(1-\alpha)(2\alpha-1)(2\alpha-3)} \times\left\{4 - \frac{6}{\alpha} - \frac{(1+\alpha)(2+\alpha)}{(1-\alpha)(2-\alpha)} + \frac{12\alpha(\gamma-1)}{(1-\alpha)(\gamma-\alpha)} + 2\frac{3-2\alpha}{\gamma+\alpha-1} + \frac{(\gamma+\alpha)(\gamma+\alpha+1)}{(\gamma-\alpha)(\gamma-\alpha+1)}\right\} + \frac{3}{4}\frac{\pi\cot\pi\alpha + \Psi(\gamma+\alpha-1) - \Psi(\gamma-\alpha)}{\alpha(1-\alpha)(2\alpha-1)(2\alpha-3)}$$

$$\mathcal{G}_{12}(n=2) = -\frac{1}{24}\left\{\frac{6}{(\alpha-2)(\alpha-1)\alpha(\alpha+1)} + \frac{3}{\alpha-\gamma} + \frac{1}{2+\alpha-\gamma} + \frac{3}{-1-\alpha+\gamma} + \frac{1}{1-\alpha+\gamma}\right\}$$

$$\mathcal{G}_{21}(n=0) = -\frac{1}{8(-1+\alpha)\alpha(-1+2\alpha)(1+2\alpha)} \times\left\{1 - \frac{6}{\alpha} - \frac{6}{1+\alpha} - \frac{2(2+\alpha)}{-1+\alpha} + \frac{6(1+2\alpha)(-1+\gamma)}{(1+\alpha)(\alpha+\gamma)} - \frac{(\alpha-1)(\alpha-8)+\gamma(\gamma-2\alpha-15)}{(-1+\alpha+\gamma)(\alpha+\gamma)} + \frac{2(1+\alpha+\gamma)}{\alpha-\gamma}\right\} - \frac{3}{4}\frac{\pi\cot\pi\alpha - \Psi(-\alpha+\gamma) + \Psi(-1+\alpha+\gamma)}{(-1+\alpha)\alpha(-1+2\alpha)(1+2\alpha)}$$

$$\mathcal{G}_{21}(n=2) = -\frac{(-1+\gamma)(3\alpha^2(1+\alpha)(5+7\alpha)+2\gamma-\alpha(6+\alpha(29+27\alpha))\gamma+(1+2\alpha)(-1+5\alpha)\gamma^2}{12(-1+\alpha)\alpha^2(1+\alpha)^2(1+2\alpha)(\alpha-\gamma)(1+\alpha-\gamma)(2+\alpha-\gamma)} - \frac{\pi\cot\pi\alpha - \Psi(-2-\alpha+\gamma) + \Psi(1+\alpha+\gamma)}{4\alpha(1+\alpha)(1+2\alpha)(3+2\alpha)}$$

D Comparison with the calculation of Evanson and Forshaw

We write here another expression for (3.12) in order to try to match at this level our calculation with the one of [10]. We perform the following transformation on the (antiholomorphic) ${}_2G_1$-functions in formula (3.12) [14]:

$${}_2G_1(\alpha,\beta,\gamma,z) = \frac{\pi}{\sin\pi(\gamma-\alpha-\beta)} \times\left\{\frac{1}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)}\,{}_2G_1(\alpha,\beta,1+\alpha+\beta-\gamma,1-z) - \frac{z^{1-\gamma}(1-z)^{\gamma-\alpha-\beta}}{\Gamma(1-\alpha)\Gamma(1-\beta)}\,{}_2G_1(1-\alpha,1-\beta,1-\alpha-\beta+\gamma,1-z)\right\}. \quad \text{(D.51)}$$

One makes use of the relations:

$$\begin{aligned} a_0+a_1+1 &= 2b_1 \\ \tilde{a}_0+\tilde{a}_1+1 &= 2\tilde{b}_1 \ . \end{aligned} \tag{D.52}$$

The result reads:

$$\begin{aligned} &-2i\pi\frac{\mu}{\sin\pi b_1}\frac{\sin\pi(b_1-a_0)}{\sin\pi b_1}\Bigg\{\alpha^{1-b_1}\bar{\alpha}^{1-\tilde{b}_1}\frac{\sin\pi(\tilde{b}_1-\tilde{a}_0)}{\Gamma(1-a_0)\Gamma(1-a_1)} \\ &\times{}_2G_1(b_1-a_0,b_1-a_1,b_1,1-\alpha) \\ &\times{}_2G_1(\tilde{b}_1-\tilde{a}_0,\tilde{b}_1-\tilde{a}_1,2-\tilde{b}_1,\bar{\alpha}) \\ &+(1-\alpha)^{1-b_1}(1-\bar{\alpha})^{1-\tilde{b}_1}\frac{\sin\pi(b_1-a_0)}{\Gamma(1-\tilde{a}_0)\Gamma(1-\tilde{a}_1)} \\ &\times{}_2G_1(b_1-a_0,b_1-a_1,2-b_1,1-\alpha) \\ &\times{}_2G_1(\tilde{b}_1-\tilde{a}_0,\tilde{b}_1-\tilde{a}_1,\tilde{b}_1,\bar{\alpha}) \\ &-\alpha^{1-b_1}(1-\bar{\alpha})^{1-\tilde{b}_1}\frac{1}{\pi}\sin^2\pi(\tilde{b}_1-\tilde{a}_0)\frac{\Gamma(\tilde{a}_0)\Gamma(\tilde{a}_1)}{\Gamma(1-a_0)\Gamma(1-a_1)} \\ &\times{}_2G_1(b_1-a_0,b_1-a_1,b_1,1-\alpha){}_2G_1(\tilde{b}_1-\tilde{a}_0,\tilde{b}_1-\tilde{a}_1,\tilde{b}_1,\bar{\alpha}) \\ &-(1-\alpha)^{1-b_1}\bar{\alpha}^{1-\tilde{b}_1}\frac{1}{\pi}\sin\pi(b_1-a_0)\sin\pi(\tilde{b}_1-\tilde{a}_0) \\ &\times{}_2G_1(b_1-a_0,b_1-a_1,2-b_1,1-\alpha) \\ &\times{}_2G_1(\tilde{b}_1-\tilde{a}_0,\tilde{b}_1-\tilde{a}_1,2-\tilde{b}_1,\bar{\alpha})\Bigg\} \ . \end{aligned} \tag{D.53}$$

In our particular case, α is real and so $\bar{\alpha}=\alpha$.

Furthermore, if one is only interested by the case $n=0$ and, say, the non-flip helicity amplitude, then $a_0=\tilde{a}_0$, $a_1=\tilde{a}_1$ and $b_1=\tilde{b}_1$. One will have to integrate this expression over α after having multiplied it by a symmetric fonction under the exchange $\alpha\to 1-\alpha$.

Hence one sees on the previous formula that the first two terms give similar contributions after integration and are proportional to the one quoted in [10], (3.46). The two other terms do not appear in the latter. So at this stage the intermediate forms differ, but the integrated results are identical. However, we failed to explain why.

E The vanishing of the $|n|>2$ components

In the core of the text, the cancellation of the components $|n|>2$ appears as an outcome of a complicated calculation, whose last step is done using `mathematica`. In this appendix, we show more explicitly how this occurs. We demonstrate more generally that for $f(\alpha)=\alpha^m+(1-\alpha)^m$, all the components $n>m$ vanish after integration over α. We restrict our calculation to the case $\delta=\tilde{\delta}$, which corresponds to the helicity-conserving amplitudes.

Our starting point is (3.17). We shall prove the vanishing of the factor:

$$\begin{aligned} &\frac{\pi}{\sin\pi(\frac{1}{2}+\delta-s)}\mathcal{J}^m(a,\frac{1}{2}+\delta-s) \\ &-\frac{\pi}{\sin\pi(\frac{3}{2}-\delta+s)}\mathcal{J}^m(1-\tilde{a},\frac{3}{2}-\tilde{\delta}+s) \\ &\quad=\frac{\pi}{\sin\pi(\frac{1}{2}+\delta-s)}\left(\mathcal{J}^m(a,c)+\mathcal{J}^m(1-\tilde{a},2-c)\right), \end{aligned} \tag{E.54}$$

which appears in the brackets in (3.17). For this purpose, we compute the $\mathcal{J}$s in terms of ${}_4G_3$-functions. The result is straightforward:

$$\begin{aligned} &\mathcal{J}^m(a,c)+\mathcal{J}^m(1-\tilde{a},2-\tilde{c}) \\ &=-{}_4G_3\left(\begin{matrix}\tilde{a},1-\tilde{a},m+2-c,1+m\\2-c,m+1+a,m+2-a\end{matrix};1\right) \\ &-{}_4G_3\left(\begin{matrix}a,1-a,m+c,1+m\\c,m+1+\tilde{a},m+2-\tilde{a}\end{matrix};1\right) \\ &+{}_4G_3\left(\begin{matrix}c-\tilde{a},c+\tilde{a}-1,m+1+c,m+c\\c,c+a+m,c-a+m+1\end{matrix};1\right) \\ &+{}_4G_3\left(\begin{matrix}2-c-a,1-c+a,m+3-c,m+2-c\\2-c,2-c+\tilde{a}+m,3-c-\tilde{a}+m\end{matrix};1\right) \end{aligned} \tag{E.55}$$

The point is that for $n>m$, the two first terms compensate, as we will prove explicitely. The same is true for the two last terms. Let us use the following identities:

$$\begin{aligned} &\int\frac{ds}{2i\pi}\cos\pi s\,\Gamma(-s)\frac{\prod_{i=0}^3\Gamma(s+a_i)}{\prod_{j=1}^3\Gamma(s+b_j)}={}_4G_3(a_i;b_j;1) \\ &\int\frac{ds}{2i\pi}\sin\pi s\,\Gamma(-s)\frac{\prod_{i=0}^3\Gamma(s+a_i)}{\prod_{j=1}^3\Gamma(s+b_j)}=0\ . \end{aligned} \tag{E.56}$$

The first one is the Mellin-Barnes representation of the hypergeometric function. The contour is a path in the complex plane for the s variable, which separates the poles of $\Gamma(-s)$ and those of the $\Gamma(s+a_i)$. Both identities are obtained by closing the contour to the right (the second identity stems from the fact that $\sin\pi s\,\Gamma(-s)$ has no pole). One can also close the contour to the left:

$$\begin{aligned} &\int\frac{ds}{2i\pi}\cos\pi s\,\Gamma(-s)\frac{\prod_{i=0}^3\Gamma(s+a_i)}{\prod_{j=1}^3\Gamma(s+b_j)} \\ &=\sum_{i=0}^3\cos\pi a_i\frac{\prod_{j=1}^3\sin\pi(a_i-b_j+1)}{\prod_{j=0\neq i}^3\sin\pi(a_i-a_j+1)} \\ &\times{}_4G_3\left(\begin{matrix}a_i,a_i-b_j+1\\a_i-a_j+1\end{matrix};1\right) \\ &\int\frac{ds}{2i\pi}\sin\pi s\,\Gamma(-s)\frac{\prod_{i=0}^3\Gamma(s+a_i)}{\prod_{j=1}^3\Gamma(s+b_j)} \\ &=\sum_{i=0}^3\sin\pi a_i\frac{\prod_{j=1}^3\sin\pi(a_i-b_j+1)}{\prod_{j=0\neq i}^3\sin\pi(a_i-a_j+1)} \\ &\times{}_4G_3\left(\begin{matrix}a_i,a_i-b_j+1\\a_i-a_j+1\end{matrix};1\right)\ . \end{aligned} \tag{E.57}$$

Identifying these two sets of equalities, we obtain:

$$
\begin{aligned}
{}_4G_3(a_i;b_j;1) &= \sum_{i=0}^{3} \cos\pi a_i \frac{\prod_{j=1}^{3}\sin\pi(a_i-b_j+1)}{\prod_{j=0\neq i}^{3}\sin\pi(a_i-a_j+1)} \\
&\quad \times {}_4G_3\left(\begin{matrix}a_i, a_i-b_j+1\\ a_i-a_j+1\end{matrix};1\right) \\
0 &= \sum_{i=0}^{3} \sin\pi a_i \frac{\prod_{j=1}^{3}\sin\pi(a_i-b_j+1)}{\prod_{j=0\neq i}^{3}\sin\pi(a_i-a_j+1)} \\
&\quad \times {}_4G_3\left(\begin{matrix}a_i, a_i-b_j+1\\ a_i-a_j+1\end{matrix};1\right) . \qquad \text{(E.58)}
\end{aligned}
$$

One sets the parameters as follows: $(a_i) = (\tilde{a}, 1-\tilde{a}, m+2-c, 1+m)$, $(b_i) = (2-c, m+1+a, m+2-a)$, so that the equalities (E.58) become

$$
\begin{aligned}
\mathcal{G}_1 &= \cos\pi\tilde{a} \times \mathcal{F}_1 + \cos\pi(1-\tilde{a}) \times \mathcal{F}_2 \\
&\quad + \cos\pi(m+1)\frac{\sin\pi(c+m)\sin\pi(1-a)\sin\pi a}{\sin\pi(m+2-\tilde{a})\sin\pi(m+1+\tilde{a})\sin\pi c} \\
&\quad \times \mathcal{G}_2 + \cos\pi(m+2-c) \times \mathcal{D} \qquad \text{(E.59)} \\
0 &= \sin\pi\tilde{a} \times \mathcal{F}_1 + \sin\pi(1-\tilde{a}) \times \mathcal{F}_2 \\
&\quad + \sin\pi(m+1)\frac{\sin\pi(c+m)\sin\pi(1-a)\sin\pi a}{\sin\pi(m+2-\tilde{a})\sin\pi(m+1+\tilde{a})\sin\pi c} \\
&\quad \times \mathcal{G}_2 + \sin\pi(m+2-c) \times \mathcal{D} , \qquad \text{(E.60)}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{G}_1 &= {}_4G_3\left(\begin{matrix}\tilde{a}, 1-\tilde{a}, m+2-c, 1+m\\ 2-c, m+1+a, m+2-a\end{matrix};1\right) \\
\mathcal{F}_1 &= \frac{\boxed{\sin\pi(\tilde{a}-a-m)}\sin\pi(c+\tilde{a}-1)\sin\pi(\tilde{a}+a-m-1)}{\sin\pi(2\tilde{a})\sin\pi(c+\tilde{a}-m-1)\sin\pi(\tilde{a}-m)} \\
&\quad \times {}_4G_3\left(\begin{matrix}\boxed{\tilde{a}-a-m}, \tilde{a}+a-m-1, \tilde{a}, c+\tilde{a}-1\\ 2\tilde{a}, c+\tilde{a}-m-1, \tilde{a}-m\end{matrix};1\right) \\
\mathcal{F}_2 &= \frac{\boxed{\sin\pi(a-\tilde{a}-m)}\sin\pi(-\tilde{a}-a-m+1)\sin\pi(c-\tilde{a})}{\sin\pi(-2\tilde{a}+2)\sin\pi(c-\tilde{a}-m)\sin\pi(1-m-\tilde{a})} \\
&\quad \times {}_4G_3\left(\begin{matrix}\boxed{a-\tilde{a}-m}, 1-\tilde{a}, c-\tilde{a}, 1-m-a-\tilde{a}\\ 2-2\tilde{a}, c-\tilde{a}-m, 1-\tilde{a}-m\end{matrix};1\right) \\
\mathcal{G}_2 &= {}_4G_3\left(\begin{matrix}m+1, a, 1-a, c+m\\ c, m+1+\tilde{a}, m+2-\tilde{a}\end{matrix};1\right) \qquad \text{(E.61)} \\
\mathcal{D} &= \frac{\sin\pi(m+1)\sin\pi(2-a-c)\sin\pi(a-c-1)}{\sin\pi(m+3-c-\tilde{a})\sin\pi(m+2+\tilde{a}-c)\sin\pi(2-c)} \\
&\quad \times {}_4G_3\left(\begin{matrix}m+2-c, m+1, 2-a-c, a-c-1\\ m+3-c-\tilde{a}, m+2+\tilde{a}-c, 2-c\end{matrix};1\right)
\end{aligned}
$$

Let us first notice that the coefficient in front of $\mathcal{G}_2$ in (E.59) is (-1), and is 0 in (E.60). Moreover, the term $\mathcal{D}$ is the product of $\sin\pi(m+1)$ times a regular function, which means that it vanishes. The system then rewrites:

$$
\begin{aligned}
&\mathcal{G}_1+\mathcal{G}_2 = \cos\pi\tilde{a} \times (\mathcal{F}_1+\mathcal{F}_2) \qquad \text{(E.62)} \\
&\mathcal{F}_1-\mathcal{F}_2 = 0 . \qquad \text{(E.63)}
\end{aligned}
$$

Let us show that $\mathcal{F}_1$ is null for $n > m$. First note that $\tilde{a}-a-m = n-m$. For $n > m$, the ${}_4G_3$-function in (E.61) is regular whereas the "boxed" sine in front of it is zero, hence $\mathcal{F}_1 = 0$ (this is not true for $n \leq m$ since then the "boxed" argument of the ${}_4G_3$-function is a negative integer, so the ${}_4G_3$-function is singular). Then (E.63) implies that $\mathcal{F}_2$ is also zero, and (E.62) leads to the identity $\mathcal{G}_1+\mathcal{G}_2 = 0$. Noticing that $\mathcal{G}_1$ and $\mathcal{G}_2$ are nothing else than the two first terms in (E.55), this completes the proof.

Let us illustrate these cancellation properties by taking the particular case $c=1$. It is of physical interest, since it corresponds to the impact factor for the $\gamma \to \gamma$ transition (real photons) which has been studied earlier in [11]. In this case, (E.55) reduces to $\mathcal{G}_1+\mathcal{G}_2|_{c=1} = 0$. But we have also we have $\mathcal{G}_1 = \mathcal{G}_2$ since $\mathcal{G}_2 = \mathcal{G}_1|_{a\leftrightarrow\tilde{a}}$ and since we showed (see the core of the paper) that the result is invariant by this substitution. Finally, we obtain, for $n > m$:

$$
{}_4G_3\left(\begin{matrix}\tilde{a}, 1-\tilde{a}, 1+m, 1+m\\ 1, m+1+a, m+2-a\end{matrix};1\right) = 0 .
$$

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